A Hybrid Regularization Algorithm for High Contrast Tomographic Image Reconstruction

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Abstract— *The common Level set based reconstruction method (LSRM) is applied to solve a piecewise constant inverse problem using one level set function and considers two different conductivity quantities for the background and the inclusion (two phases inclusion). The more the number of the piecewise constant conductivities in the medium, the higher the calculation effort of the LSRM, using multiple level set functions, will be. The assumption of piecewise constant conductivities (coefficients) is to discriminate between two regions with sharp conductivity interface; however, it may not be a realistic assumption when there are smooth conductivity gradients inside each region as well. In this paper, we propose a hybrid regularization method (HRM), which is a two steps solution, to solve ill-posed, non-linear inverse problem with smooth conductivity transitions. The first step of this hybrid inversion framework plays the role of an initializing procedure for the second step, and acts in a similar way as a* source type *inversion method. In the first stage, the LSRM with one level set function is applied to determine the region of interest, which is defined as the region with sharpest interface. Then in the second stage, an inverse solver with penalty terms based on sum of absolute values (L1 norms), which are highly robust against measurement errors, is applied to reconstruct the conductivity changes inside the determined ROI. The generated forward solution in the final iteration of the level set is fed to the second stage where the L1 norms based penalty terms are minimized using primal-dual interior point method (PDIPM). The PDIPM has been shown to be effective in minimizing the L1 norms. The reconstructed images with the proposed HRM maintains the edge information as well as the smooth conductivity variations, a trait absent in all previously established level set based reconstruction method. The integration of the LSRM and the PDIPM can generate less noisy reconstructed images when comparing either with the reconstruction results of the PDIPM or with those of squared error based reconstruction methods, such as Gauss-Newton (GN) method. Our proposed HRM is tested on a circular 2D phantom with either sharp conductivity gradients (piecewise constant coefficients) or smooth conductivity transitions (smooth coefficients). We show that the proposed HRM maintains sharp edges and is robust against the measurement noise.*

Keywords: Inverse problem, tomographic image reconstruction, primal-dual interior point method, level set, sum of absolute values (L1 norms), and electrical impedance tomography

1. Introduction

Tomography has found widespread applications in many scientific fields, including physics, chemistry, astronomy, geophysics, and medicine. Tomographic image reconstruction is a visualization technique to produce a cross sectional image of the internal structures of an object. Different physical quantities are measured in different tomographic imaging modalities. For instance, X-ray CT measures the number of x-ray photons transmitted through the patient along individual projection lines and reconstructs the distribution of the linear attenuation coefficients in the desired cross sectional slice. To be able to easily differentiate between variant tissue types, high contrast image reconstruction is highly demanded in tomography. In this paper, we propose a novel high contrast image reconstruction algorithm to produce high quality reconstructed images. To show the implementation of the proposed reconstruction algorithm, we apply electrical impedance tomography (EIT).

EIT is a non-invasive tomographic imaging modality which reconstructs the internal conductivity distribution using the measurement of several difference voltages collected from the electrode pairs attached at the surface of the medium. Electrical current is injected into the medium and the resulting difference voltages at the surface of the medium is measured using several electrodes. The conductivity distribution is then estimated based on the measured voltages and medium geometry.

EIT image reconstruction is an ill-defined inverse problem. The possible number of surface measurements is limited creating a highly under-determined system with low spatial resolution. To address the instability of the EIT images, there are different regularization techniques to introduce priori information into the reconstruction algorithm. The traditional priori information is to have smooth conductivity gradients inside the medium. The assumption of smooth conductivity changes creates blurred reconstructed images. However, the physiologically meaningful reconstructed images are created when we consider sharp conductivity changes between organ boundaries. There are several reconstruction techniques to maintain sharp edges. Dai and Adler (2008) used a weighted identity matrix (LDM) for the regularization [2]. They assume piecewise smooth conductivity changes in the medium and use total variation (TV) technique to suppress the background fluctuations. Borsic (2010) compared the LDM technique and the Primal Dual-Interior Point Method (PDIPM) [1]. Borsic and Adler (2012) show the superiority of the L1 norm usage when comparing with L2 norm [3]. They use the L1 norm, on either the regularization or the data term of an inverse problem to produce higher quality reconstructed images when comparing with reconstructed images of applying L2 norm. We show the first application of level set based reconstruction method (LSRM) to produce clinically useful images by preserving the edges [4]. The LSRM is a nonlinear inversion scheme using L2 norm on the data and the regularization term. The implementation of the LSRM is based on the Gauss-Newton (GN) optimization approach to iteratively reduce a given cost functional, which is the norm of the difference between the simulated and measured data. In comparison to the voxel based reconstruction method (VBRM), the LSRM has the advantage of introducing the conductivity of background and that of inclusions as known priori information into the reconstruction algorithm, enabling it to preserve the edges and to provide sharp contrasts.

In this paper, we formulate a hybrid regularization method (HRM) containing the advantage of using the common LSRM in tracking propagating interfaces, preserving the edges, and that of using L1 norm in precisely reconstructing the conductivity variations inside the medium. We introduce the PDIPM solver into the level set based GN optimization algorithm (figure 1). We tested the proposed HRM using a circular 2D phantom under different test conditions: 1) without added noise. 2) with added zero-mean Gaussian noise (-60dB). 3) with noise (-60 dB) and data outliers (one measurement out of 208 for every EIT frame was missed). The 2D phantom is used in two different scenarios: 1) when there are sharp inclusions in the upper and lower regions of the phantom, 2) when there are smooth conductivity changes when traveling from the inclusion boundary towards the center of the inclusion. We compare the results of the proposed HRM with two the state of the art reconstruction algorithms (the PDIPM with L1 norms on the inverse problem terms and the GN approach)over the same simulated data and test condition.

2. Methodology

The idea of the HRM is similar to the idea of a *source-type* inverse scheme, where a nonlinear inverse problem is divided into two stand-alone subproblems and each subproblem is solved separately [6]. The advantage of *source-type* inverse method is its low sensitivity to the nonlinearity of the inverse problem. In the first stage of the proposed HRM, a *equivalent source* which fits the data (forward solution), with the assumption of known background and inclusion conductivity, is produced. The equivalent source is an approximation of the final inverse solution. Assuming the known background and inclusion conductivities, the LSRM is applied in the first stage of the HRM. The inverse solution of the LSRM is an approximation of the final solution and is defined as region of interest (ROI), the region with sharpest interface enclosing smooth conductivity transitions. In the second stage of the HRM, a PDIPM solver is applied to solve the second subproblem with smooth conductivity transitions defined over the achieved ROI from the first stage. In the second stage, PDIPM solver with penalty terms based on sum of absolute values (L1 norms), which are highly robust against measurement errors, is applied to reconstruct the smooth conductivity changes inside the determined ROI. The generated forward solution in the first stage, achieved in the final iteration of the level set, is fed to the second stage where the L1 norm based penalty terms are minimized using the PDIPM. The PDIPM has been shown to be effective in minimizing the L1 norms [3]. In the following, we introduce the applied LSRM. For details about the PDIPM framework, we refer the reader to [3].

The LSRM has been shown the capability of being suitable for reconstructing object with fast changes at its interface over time [5], [4]. The classic formulation of the LSRM assumes that the reconstructed image can take only two conductivity values: one for background with value σ_b and another one for inclusions with value σ_i. The regions which form the background and the inclusions are defined by a level set function (LSF), Ψ, which is a signed distance function to identify the unknown interface between the two high contrast regions. The value of the LSF is zero on the interface, negative inside the interface, and positive outside. A detailed study of the LSRM is shown in [4].

To begin with the HRM, we need to define an initial LSF, which may be a circle on level zero; and then deform this initial LSF using a predefined energy functional iteratively. Figure 1 represents the steps as k represents the iteration number. After defining the initial LSF, the mapping function Φ projects the LSF to a 2D mesh to be fed to difference solver block to calculate the system sensitivity matrix, Jacobian (J_k). To update the energy functional of the LSRM, ΔLSF_k, the element differential potential vectors, Δd_i is calculated. The initial LSF is then deformed by ΔLSF_k generating a new LSF. This new LSF is fed again to difference solver block for another iteration if the current iteration number (k) is not bigger than a maximum iteration number (K). According to the chain rule, the level set (LS) sensitivity matrix can be written as below:

$$\begin{aligned} Sensitivity = J_{LS} = \frac{\partial d}{\partial \Psi} \quad &= (\tfrac{\partial G}{\partial \Phi(\Psi)})(\tfrac{\partial \Phi(\Psi)}{\partial \Psi}) \\ &= (J_{GN})(M), \end{aligned} \tag{1}$$

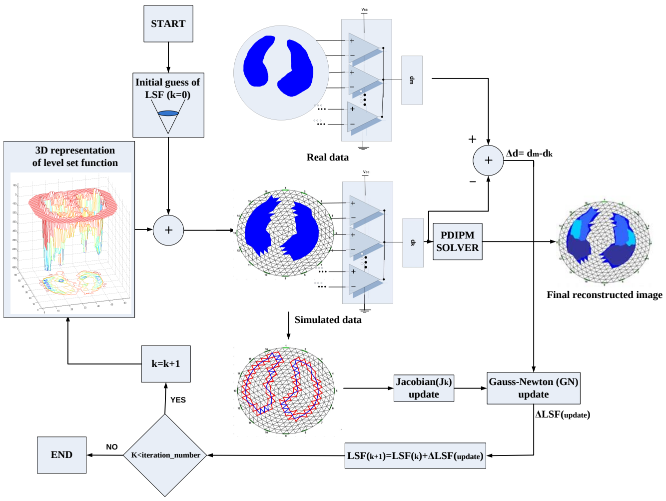


Fig. 1: The hybrid regularization technique using difference solver. Steps from top to down: LSF initial guess, inverse difference solver, PDIPM solver, Guass-Newton update, LSF displacement by the given update, and iteration number increment.

where $\frac{\partial G}{\partial \Phi(\Psi)}$ stands for the traditional GN sensitivity matrix (J_{GN}), and $\frac{\partial \Phi(\Psi)}{\partial \Psi} = M$ is the matrix representing the mapping function $(\Phi(\Psi))$. Then, the new GN update is [4]

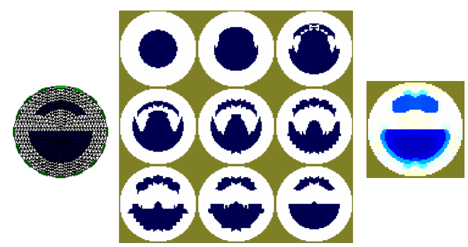
$$\Psi_{k+1} = \Psi_k + \lambda \Big[(J^T_{(LS,k)} J_{(LS,k)} + \alpha^2 L^T L)^{-1} \times (J^T_{(LS,k)}(d_{real} - d(\Psi_k))) - \left[\alpha^2 L^T L(\Psi_k - \Psi_{int})\right] = \Psi_k + GN_{update} = LSF(k) + \Delta LSF, \quad (2)$$

where Ψ_{int} in the update term corresponds to the initial estimate of the LSF. The length parameter λ determines the magnitude of the LSF displacement, changing the shape of inclusion, in a given update. The higher the λ, the higher the LSF displacement will be. The effect of the regularization parameter α depends on the choice of the regularization operator L. As α increases, the smoother the final LSF tends to be.

In the second stage of the HRM, the forward solution of the LSRM, which is the differential potential vectors of the simulated data (equation (3) in the Appendix A), is fed to the PDIPM solver. The PDIPM iteratively minimizes the norms (L1 or L2 norms) on the data and the regularization terms of the inverse problem, see [3]. The inverse solution of the PDIPM is the smooth conductivity changes inside the ROI, defined by the final evolution of the level set function at final iteration K. Plugging the initial forward solution, measured from the achieved ROI from the first stage, into the PDIPM formulation, the convergence of the PDIPM happens rapidly.

3. Simulated data

We used 16 electrodes on one electrode plane with a circular finite element model. The adjacent current stimulation was considered for the evaluation of our simulation. Figure 2(a) shows the used 2D phantom to generate simulated data with 1024 elements. The phantom contains two sharp inclusions with the same conductivity located in the upper and the lower part of the mesh. The background conductivity value is 1 S/m and the inclusions have the conductivity of 0.9 S/m. The inverse problem used the mesh density of 576 elements, which was different than the mesh density of the forward problem (1024 elements). Figure 3(a) simulates the smooth conductivity changes in a circular, low conductive inclusion. The conductivity gradually decreases when traveling from the most outer band towards the inner. The most outer band has the highest conductivity of 0.9 S/m. The conductivity decreases smoothly with a small step size of 0.1 S/m as the distance to the center of the inclusion decreases. The most inner circle has the lowest conductivity of 0.4 S/m.



(a) (b) (c)

Fig. 2: The reconstructed image using the proposed hybrid regularization technique with difference solver. (a) The 2D phantom applied to generate the simulated data. (b) The reconstructed images of the level set based reconstruction method at every iteration. (c) The final reconstructed image using the proposed hybrid regularization method at iteration 9.

4. Results

The performance of the proposed HRM was assessed over three different test conditions: 1) original numerical phantom, 2) noisy phantom with an added zero mean Gaussian noise bringing the SNR to the typical value of 60 dB. 3) data outliers plus noise. Figure 2 shows the simulated result using the proposed hybrid regularization method. Figure 2(a) is the applied 2d phantom. The reconstructed image of the LSRM in each iteration is shown in figure 2(b). Figure 2(c) shows the final reconstructed image from the proposed HRM. The L1 norms on both the data term and the regularization term are applied to achieve the result presented in figure 2(c). The ROI is defined by the LSRM (the lower panel on the right corner in figure 2(b)) and the conductivity values inside the ROI are approximated using the PDIPM solver in figure 2(c). HRM image shows sharp edges at the interface as well as the conductivities inside the ROI, the region segmented by the level set technique. The reconstructed image by the proposed hybrid technique is very similar to the real phantom, with almost no artifacts at the electrode boundary. Figure 3 shows

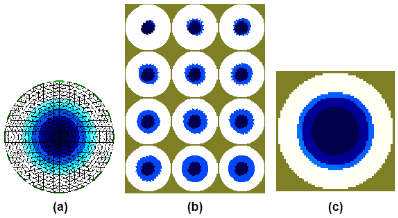


Fig. 3: The reconstructed image using the proposed hybrid regularization technique over EIT simulated data with smooth conductivity transitions. (a) The 2D phantom applied to simulate a low conductive inclusion with smooth conductivity changes. (b) The reconstructed images of the level set based reconstruction method with *two level set functions* at every iteration. (c) The final reconstructed image using the proposed hybrid regularization method at iteration 12.

the reconstruction results of the HRM for EIT simulated data with smooth conductivity changes. The conductivity values decrease as a function of radius in figure 3(a). In the upper panel on the left corner in figure 3(b), the LSRM with *two level set functions* divides the image plane into 4 regions, shown in light blue, dark blue, red, and white (background). The iteration of the LSRM with two level set functions are depicted in figure 3(b). The convergence is achieved after 12 iterations and the ROI is determined in the lower panel on the right corner in figure 3(b). Figure 3(c) shows the final reconstructed image of the HRM, which approximates the smooth conductivity changes inside the determined ROI. Figure 4 compares the HRM with two the state of the art EIT reconstruction methods ($PDIPM_{L2L2}$ and $PDIPM_{L1L1}$). In figure 4, a set of simulated results using PDIPM framework for L2 norm on the data mismatch term and the L2 norm on the regularization term (L2L2 problem) as well as for L1L1 problem under 3 different test conditions is represented. The row (a) is when there is no added noise to EIT simulated data. The row (b) is when we add zero-mean Gaussina noise (-60 dB) to EIT simulated data, and the row (c) is when there are both added noise and data outliers. As it can be seen in figure 4, there are acceptable reconstructed images when there is no noise (row (a)). However, the quality of the reconstructed images drops with the added noise (row(b)). The HRM results offer lower sensitivity to added noise (the last two panels in row (b)) when comparing with PDIPM results (the first two panels in row (b)). The reconstruction quality noticeably drops in row (c) for the PDIPM when there are data outliers and measurement noise. The HRM results are slightly less sensitive to added noise and data outliers, when comparing with the results of the PDIPM. The hyperparameter selection for all the results in figure 4 was based on trial and error over a wide range of values including very small quantities to very big ones. For each method in figure 4, the best hyperparameter with lowest residue error (mismatch term between real conductivity and the approximated conductivity) was selected.

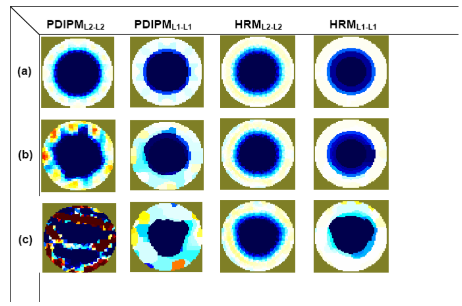


Fig. 4: EIT Reconstructed images for the 2D phantom in figure 3(a) using the proposed hybrid regularization technique as well as PDIPM algorithm with L2L2, or L1L1 norms under 3 different test conditions. (a) without any added noise to the simulated data. (b) with added zero-mean Gaussian noise (-60 dB). (c) with the presence of data outliers and noise.

5. Discussion and conclusion

We formulate a hybrid regularization method in difference mode to address the instability of the EIT reconstruction method. The concept of the proposed HRM is similar to the idea of a *source-type* inverse scheme, where a nonlinear inverse problem is divided into two stand-alone subproblems and each subproblem is solved separately. The proposed two step solution makes the final inverse solution of the HRM less sensitive to non-linearity of the problem, which is the notion of *source type* methods. In the first stage of the HRM, we approximate the inverse solution using the level set method and define a region of interest with sharpest interface, containing smooth conductivity transitions. Then in the second stage, the PDIPM solver is utilized to reconstruct the smooth conductivity changes inside the ROI. The main advantage of the proposed HRM is that it monitors both the smooth conductivity changes inside the inclusion as well as the big conductivity changes at the interface between the inclusion and the background (figure 3) with minimum amount of electrode artifacts at the medium boundary. The test studies based on Figure 4 show that PDIPM$_{L2L2}$ is sensitive to the measurement noise as well as the data outliers (the first column of figure 4). The PDIPM$_{L1L1}$ is better robust against the measurement noise (row (b) in figure 3). The HRM$_{L2L2}$ is not severely affected by the measurement noise (row (b) in figure 4); however, is sensitive when there is both outliers and noise (row (c) in figure 4). The HRM$_{L1L1}$ shows better performance in two scenarios: when there is no noise (row (a) in figure 4) , and with added zero-mean Gaussian noise (row (b) in figure 4).

6. Appendix A

The nonlinear error function with L2 norms over the data fidelity and the regularization terms can be written as follows:

$$e = \|y - F(\Psi(x))\|^2 + \|\Phi(\Psi(x)) - x_0\|^2, \tag{3}$$

where,

$$F(\Psi(x)) = G(\Phi(\Psi(x))), \tag{4}$$

function F maps electrical conductivity distribution $\Phi(\Psi(x))$ to the measured data, G is system matrix, and $\Psi(x)$ is the level set function. To minimize the nonlinear error function, we take the first derivative of the error function with respect to $\Psi(x)$:

$$\begin{aligned}\frac{de}{d\Psi(x)} = \frac{d}{d\Psi(x)}[(y - F(\Psi(x)))^t(y - F(\Psi(x))) + \\ (\Phi(\Psi(x)) - x_0)^t R(\Phi(\Psi(x)) - x_0)] = \\ \frac{d}{d\Psi(x)}[y^t y - 2y^t F(\Psi(x)) + F(\Psi(x))^t F(\Psi(x)) + \\ (\Phi(\Psi(x)) - x_0)^t R(\Phi(\Psi(x)) - x_0)] = 0,\end{aligned} \tag{5}$$

We define:

$$J_{LS} = \frac{\partial F(\Psi(x))}{\partial \Psi(x)} = (\frac{\partial G}{\partial \Phi(\Psi(x))})(\frac{\partial \Phi(\Psi(x))}{\partial \Psi(x)}) = (J)(M), \tag{6}$$

we indicate $F(\Psi(x)) = F$, $\Phi(\Psi(x)) = X$, thus:

$$\begin{aligned}\frac{de}{d\Psi(x)} = \\ -2(JM)^t y + 2(JM)^t F(\Psi(x)) + 2M^t R(\Phi(\Psi(x)) - x_0) = 0, \\ \frac{de}{d\Psi(x)} = -M^t J^t y + M^t J^t F + M^t R(X - x_0) = 0, \\ M^t J^t F + M^t R(X - x_0) = M^t J^t y,\end{aligned} \tag{7}$$

we have $F = G(X)$, therefore:

$$\begin{aligned}M^t J^t G(X) + M^t R X &= M^t J^t y + M^t R x_0, \\ J_{LS}^t G(X) + M^t R X &= J_{LS}^t y + M^t R x_0,\end{aligned} \tag{8}$$

The solution, X, of (8), which is a nonlinear equation, is successively approximated during the iterations of the level set function $\Psi(x)$. The simplest approach to estimate X is to apply the common GN solver in every iteration of the level set function. In this case, we can approximate that:

$$X = (J^t J + R)^{-1} J^t y, \tag{9}$$

where R is the Tikhonov regularization matrix. The final forward solution of the LSRM (equation (4)) is fed as initial forward solution to the second stage of the proposed HRM. In the second stage, the L1 norms of data fidelity and regularization terms of the inverse problem are minimized using PDIPM framework, see Borsic and Adler (2012) for the details.

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