Optimizing Japanese Domestic Airlines Network by Evolutionary Computation

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Abstract - *In recent years, various networks have come to exist in our surroundings. Not only can the internet and airline routes be regarded as networks; protein interactions are also networks. A network is defined as a structure of nodes (points) and links (lines). As described in this paper, an airline network is used as an example of an "economic network design problem." For the airline network, modeled based on a connection model proposed by Jackson and Wolinsky, a utility function can be defined as the sum of profits obtained from each route. Furthermore, an optimization simulation using the evolutionary computation is presented for a domestic airline in Japan.*

Keywords: Airline Network, Simulation**,** Network Game Theory, Evolutionary Computation

1 Introduction

In recent years, the Japanese aviation industry has been in dire straits, and the failure of Japan Airlines (JAL) has been well publicized. Although JAL has announced abolition of one loss-producing line one after another, All Nippon Airlines (ANA), which had been expected to replace JAL, has shown bad financial health. In such a situation, finding optimal airline networks that maximize profits is extremely important. Because the profits of other routes might also be affected by abolishing a loss-making line, it is necessary to consider transitions in an aviation network. Properly speaking, when abolition of a route is determined, it is desirable to consider not only the influence of the direct flight, but the concomitant increase and decrease of passenger traffic by effects on transit passengers. Therefore, this paper defines a profit function that regards the influence on transit passengers explicitly. We seek the optimal airline network that maximizes profits. In modeling of the airline network, connections models of Jackson and Wolinsky (1996) are extended to a model in which profits generate on a link and the utility function of the network is defined.

In the paper "airline network optimization problems," analyses were conducted for the network to maximize network utility, but finding optimal network theory is difficult when assessing real-world problems. Therefore, we propose a solution using evolutionary computation to resolve "airline network optimization problems." Furthermore, optimization simulation using evolutionary computation is demonstrated for a domestic airline in Japan. Optimal networks obtained when only the rate of the passenger discount by the transit was changed were found using the proposed algorithm.

The remainder of this paper is organized as follows. Section 2 formulates an "airline network optimization problem." Section 3 presents a description of a method of an "airline network optimization problem" using an evolutionary computation. Simulation results are presented in Section 4. Finally, we state the important conclusions in Section 5.

2 Airline network

An airline network is modeled in this paper based on Jackson and Wolinsky's (1996) connections model. In Jackson and Wolinsky (1996), the form that a stable network takes is analyzed under the situation in which formation of the link with a new each node (player)(relation) and an existing link disconnection are selected in the strategy. The network formation game theory proposed by Jackson and Wolinsky (1996) can become a substantial framework when dealings between varieties of economic agents are analyzed. For example, the conclusion distribution of the Free Trade Agreement in an international trade is foreseen, and it is applied to the analysis of the decision of the best airline line network etc. In the following, after first describing the utility function in the connections model, the airline network is modeled.

2.1 Connections model

As described in this paper, an economic network is denoted as a non-directed graph $G = (V, E)$ according to the graph theory. Graph G consists of node set V and link (edge) sets E. For example, a node in a graph represents an economic player (individual, group, city, and nation), and the link stands for a transport link, a telecommunication net, an economic regional alliance, etc. A link $e \in E$ is sets of node pairs $e = \{i, j\}$. A link between i and j is simply denoted ij. Considering a complete graph that consists of node set V, an economic network that consists of nodes (players) is subgraph G of complete graph K. The values of w_{ij} is an intrinsic value to obtain node i from node j by a direct link, and c_{ij} is the running cost of link ij. The utility function to obtain node i in graph G is defined as shown below.

$$u_i(G) = \sum_{j=1|i\neq j}^{|V|} \delta^{s_{ij}(G)} w_{ij} - \sum_{j=1|ij\in G}^{|V|} c_{ij} \qquad (1)$$

|V|: number of nodes in V
δ: decay rate of the gain
$s_{ij}(G)$: shortest path length in G
w_{ij}: gain to obtain node i from node j through link ij
c_{ij}: running cost of link ij

Furthermore, $\delta \in [0,1]$ is the decay rate of the gain. Considering the shortest path length s_{ij} between ij, $\delta^{s_{ij}(G)} w_{ij}$ is the gain to obtain node i from node j through the shortest path s_{ij}. Only when the link exists directly between ij is the link running cost c_{ij} needed. The graph value (utility of the entire network) is a summation of utilities of all nodes that exist in the network. The graph value is defined by the following equation.

$$u_{net}(G) = \sum_{i=1}^{|V|} u_i(G) = \sum_{i=1}^{|V|} \sum_{j=1|j\neq i}^{|V|} \delta^{s_{ij}(G)} w_{ij} - \sum_{i=1}^{|V|} \sum_{j=1|ij\in G}^{|V|} c_{ij} \tag{2}$$

The economic network design problem above can be formulated as follows.

$$\arg\max_{G \subseteq K_{|V|}} u_{net}(G) \tag{3}$$

2.2 Airline network model

There was a problem of lacking concreteness, although the connections model of Jackson and Wolinsky (1996) was able to assume various networks. Therefore, the analytical object is focused on the airline network. In the following, the airline network is modeled based on a connections model. The point in which the airline network model differs greatly from the utility function of connections model is to examine the utility obtained from a link.

In the airline network model, a node and a link respectively signify an airport and a route. The airline route with only the outward or homeward journey is a rare case. Therefore, the airline network is assumed to be a non-directed graph that does not incorporate the direction of the connection of the link. A link $e \in E$ of non-directed graph is a set of node pairs $e = \{i, j\}$ ($i, j \in V$ and $i \neq j$) without the order. A link between i and j is denoted simply as ij, and $ji = ij$. Furthermore, r_{ij} is an income of link ij; c_{ij} is an operation cost of link ij. The profit obtained between links ij in graph G is defined as

$$\pi_{ij}(G) = \begin{cases} r_{ij} - c_{ij} & if \quad ij \in G \\ \delta^{s_{ij}(G)-1} r_{ij} & otherwise \end{cases} \tag{4}$$

|V|: number of nodes (airport)
δ: decay rate of the profit
$s_{ij}(G)$: shortest path length in G
r_{ij}: income between i and j
c_{ij}: operation cost of link ij

Also, $\delta \in [0,1]$ is the decay rate of the profit. Considering that the shortest path length s_{ij} between ij, $\delta^{s_{ij}(G)-1} r_{ij}$ is the income got from node i by a passenger who goes to node j. Only when a direct flight exists between ij is operation cost c_{ij} needed. For these analyses, it is assumed that operation cost c_{ij} is necessary only for the direct flight's existing according to connections model. It is assumed that a flight need not be increased even if the number of passengers increases by an indirect link. Consequently, for aircraft that are not used over capacity, the load factor is at the 60% level also for the main route.

In addition, the income is the product of ticket price p_{ij} and the number of passengers q_{ij}. Also, δ is the product of the decay rate of ticket price δ_1 and the passenger decay rate δ_2. Equation (4), showing the price of the airline ticket and the number of passengers, can be rewritten as the income as follows.

$$\pi_{ij}(G) = \begin{cases} p_{ij} q_{ij} - c_{ij} & if \quad ij \in G \\ \delta_1^{s_{ij}(G)-1} p_{ij} \cdot \delta_2^{s_{ij}(G)-1} q_{ij} & otherwise \end{cases} \tag{5}$$

δ_1: decay rate of ticket price
δ_2: passenger decay rate
p_{ij}: ticket price between i and j
q_{ij}: passengers between i and j

Also, $\delta_1 \in [0,1]$ is the decay rate of the ticket price. The fare that a passenger must pay indeed gives a discount if the number of times of connection increases. In addition, $\delta_2 \in [0,1]$ is the passenger decay rate. Whenever the number of connections to the destination increases by δ_2, it is included in the model that the number of passengers decreases.

Next, operation cost c_{ij} is defined. Actually, c_{ij} is the operation cost of one year for the route between i and j, and the cost function is defined by the following equations.

$$c_{ij} = OP_{ij} \cdot FY_{ij} \tag{6}$$

OP_{ij}: cost per flight
FY_{ij}: number of annual flights

$$OP_{ij} = W(d_{ij} \cdot FU) + B \tag{7}$$

W: change of the weight by the number of passengers
d_{ij}: straight line distance between i and j
FU: fuel cost per km
B: airport landing fee

$$FU = LMX(FP + TAX)/RMX \tag{8}$$

LMX: maximum fuel capacity of the aircraft
FP: price per liter of jet fuel
TAX: fuel tax per liter
RMX: longest cruising range of aircraft

$$W = \frac{WE + PF_{ij} \cdot PW}{WE + PMX \cdot PW} \tag{9}$$

WE: operating empty weight of the aircraft
PF_{ij}: number of passengers per flight
PMX: number of seats of aircraft

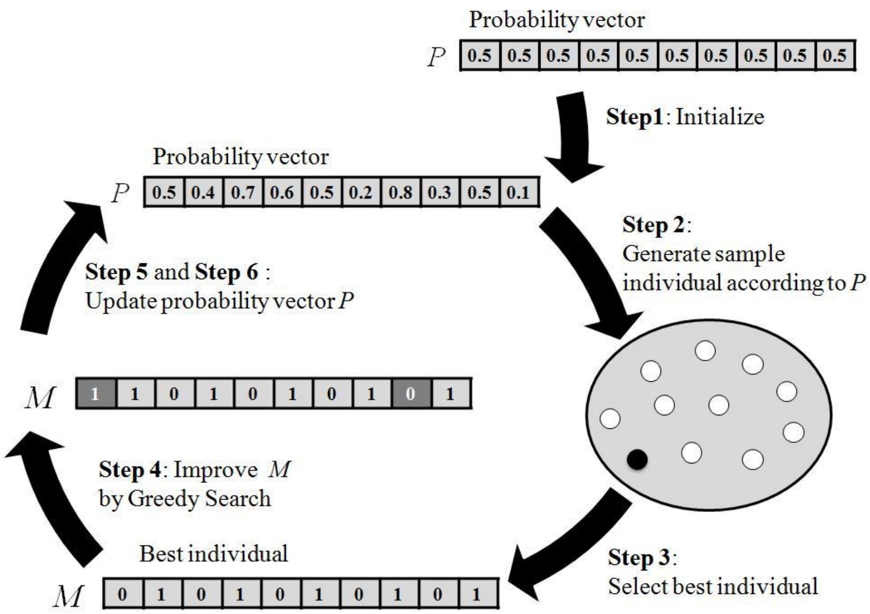

Fig. 1. Schematic diagram of algorithm.

PW: weight per passenger

The graph value $\pi_{net}(G)$ (profit of the entire airline network) is the summation of profits of all links in the network. The graph value $\pi_{net}(G)$ is defined by the following equation.

$$\pi_{net}(G) = \sum_{i=1}^{|V|}\sum_{j=1}^{|V|}\pi_{ij}(G) = \sum_{i=1}^{|V|}\sum_{j=1|j\neq i}^{|V|}\delta^{s_{ij}(G)-1}r_{ij} - \sum_{i=1}^{|V|}\sum_{j=1|ij\in G}^{|V|}c_{ij} \tag{10}$$

In equation (4), the profit when the direct link (direct flight) exists between *ij* differs when the direct link does not exist. However, no problem as $\delta^{s_{ij}(G)-1}r_{ij}$ exists because the shortest path length $s_{ij}(G)$ = 1 when a direct flight exists. Equation (10) and equation (2) are the same structures if it is excluded that the exponent of δ is $s_{ij}(G)$-1. It is understood that the airline network model is a pure application of the connections model.

The airline network optimization problem above can be formulated as follows.

$$\arg\max_{G\subseteq K_{|V|}} \pi_{net}(G) \tag{11}$$

3 Evolutionary computation for “airline network optimization problem”

It is extremely difficult to analyze the network where the utility of the entire network is maximized in non-symmetric node (player) theoretically. One kind of evolutionary computation method, Population-Based Incremental Learning (PBIL) with a local search for the approximate solution method, is applied. Optimization of the network is tried. The basic operation of the improved algorithm is almost identical to that of the usual PBIL, but there is a difference in the update process of the probability vector. The difference point is to do a greedy search based on the selected excellent individual before the probability vector is updated. The former excellent solution is replaced if a better solution is found from the former excellent solution as a result of a greedy search. However, a local search examines the range of Hamming distance 1 as the neighborhood. This proposed algorithm is called G-PBIL. In the following, the schematic diagram of algorithm is presented in Fig. 1, and detailed processing of each Step is described.

Step 1: Initialization of probability vector $\vec{P}$

The probability vector $\vec{P} = (p_1, p_2, \ldots, p_{nbit})$ is the probability that each bit of the gene becomes 1. When the search begins, the probability vector is set to all 0.5. *nbit* is the gene length, which changes according to the scale of the problem. The probability vector in *t* generation (cycle) is written as $\vec{P}^t = (p_1^t, p_2^t, \ldots, p_{nbit}^t)$.

Step 2: Generate the sample population according to probability vector $\vec{P}$

Each individual is expressed by the bit string of 0 or 1 that is called a gene, and the probability vector is simply a description of the appearance probability of 1 by the vector.

Step 3: Evaluate the population, and select an excellent individual

The population is evaluated, and an excellent individual with the best fitness in the population is selected. The selected excellent individual is the notation

$M^t = \left(m_1^t, \ldots, m_{nbit}^t\right)$. A term called *fitness* is used because the right and wrong of an objective function values change with a minimization problem or maximum problem.

For a minimization problem, it is considered that a smaller objective function value has higher fitness.

Conversely, for a maximization problem, a greater objective function value indicates higher fitness.

Step 4: Greedy search based on a selected excellent individual.

First, the population that changes the gene of M^t by only one bit is generated. Next, fitness of the population that newly generates it is calculated, and the individual with the best fitness among former M^t and newly generated populations is new M^t. Greedy search is stopped if former M^t has best fitness. Otherwise, greedy search is tried again based on new M^t. However, when trying greedy search again, changing a bit again that has already changed from M^t from the first received from PBIL is forbidden. This rule limits the frequency of a greedy search. Even if it is the maximum, the frequency of a greedy search under this rule is gene length. The reason to adopt such a rule is that it is thought that the frequency of a greedy search becomes every high if the bit is changed unrestrictedly.

Step 5: Update probability vector

The probability vector is updated using excellent individual M^t improved by a greedy search.

$$p_i^{t+1} = (1.0 - LR)\, p_i^t + LR \cdot m_i^t \tag{12}$$

LR stands for the learning rate. When *LR* is large, the search will converge rapidly to the generation's excellent individual. To evade the initial convergence, *LR* should be set to a small value. However, because a generation number required for search increases when the value of *LR* is small, setting it to an extremely small value hinders the search.

Step 6: Mutation

Mutation occurs at a constant mutation probability, and the value of each element of the probability vector updated with Step 5 is changed further according to the following equation.

$$p_i^{t+1} \leftarrow (1.0 - MR)\, p_i^{t+1} + rand \cdot MR \tag{13}$$

Mutation Rate (*MR*) is a degree of the change by the mutation. The probability vector changes greatly by *MR* large. $rand \in \{0,1\}$ is a uniform random number.

Step 7: Repetition of Step 2 – Step 6

The processing of Step 2 – Step 6 is repeated until the termination condition is satisfied. It is defined as the first generation to repeat the processing of Step 2 – Step 6 once in PBIL. The termination condition of processing when the set number of generations is passed or the convergence of the search is admitted by the convergence criterion is usually adopted as a termination condition.

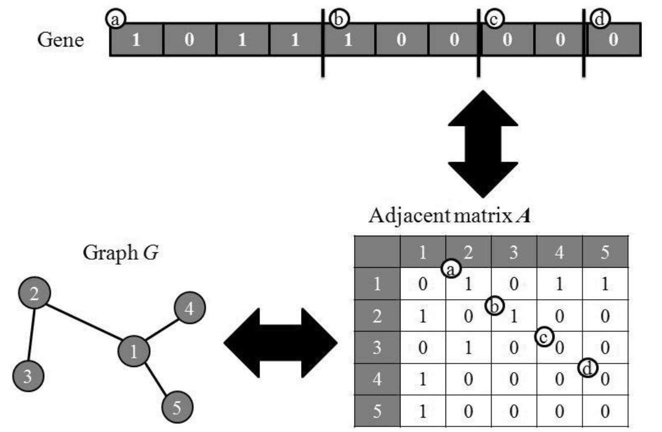

Fig. 2. Coding for a network graph with 0–1 design variable.

When this algorithm is adapted to the "network optimization problem" for which a design variable takes the discrete value of 0–1, it is necessary that a gene correspond to a graph as presented in Fig. 2. Denoting a graph by an adjacent matrix, and also making an adjacent matrix correspond to a genotype can express a graph with a gene. The adjacent matrix *A* of a graph *G* (airline network) is the matrix of |V|×|V|, where a_{ij} represents element of row *i* and column *j* of matrix *A*. It is assumed that a_{ij} =1 when the link (edge) exists directly between vertices *i* and *j*. It is also assumed that a_{ij} =0.

$$a_{ij} = \begin{cases} 1 & if \quad ij \in G \\ 0 & otherwise \end{cases} \tag{14}$$

When G-PBIL is adapted to the "network optimization problem" in which a design variable takes the discrete value of 0–1, the gene length is set as $nbit = |V|(|V|-1)/2$. In Step 3 of an algorithm, as presented in Fig. 2, a gene is changed to a graph, and the graph value is computed using equation (10). Making a computed graph value into the goodness of fit of a gene is synonymous with optimizing a graph to optimize a gene.

4 Simulation analysis of the optimal airline network

Here, the proposed algorithm is used and simulated. The simulation is targeted to 19 airports of Japan with domestic routes serving 1.5 million passengers or more annually. Figure 4 shows the existing network among the 19 airports.

4.1 Data

To conduct a simulation, some data are necessary: the ticket price p_{ij}, number of potential passengers q_{ij}, and straight line distance d_{ij} between *i* and *j*. First, we consider the ticket price p_{ij} between *i* and *j*. If between *i* and *j* is an existing route, p_{ij} examines the price of the airline ticket. However, the price cannot be examined about the route that does not exist. Then, the airline ticket price of the non-existent route is estimated by

TABLE I
Allocated populations of 19 airports (Unit: 10,000 people)

Airport name	Airport code	*pop*
1. Haneda	HND	3302.30
2. New Chitose	CTS	46.73
3. Osaka	ITM	1422.54
4. Fukuoka	FUK	312.09
5. Naha	OKA	134.70
6. Chubu	NGO	980.25
7. Kagoshima	KOJ	213.79
8. Kansai	KIX	188.20
9. Kumamoto	KMJ	184.38
10. Miyazaki	KMI	66.07
11. Hiroshima	HIJ	225.93
12. Sendai	SDJ	220.16
13. Kobe	UKB	337.01
14. Matsuyama	MYJ	122.93
15. Nagasaki	NGS	147.90
16. Komatsu	KMQ	98.09

the following regression by making the distance d_{ij} into an explanatory variable.

$$p = \alpha + \beta d \tag{15}$$

A single regression analysis that uses a straight line distance based on Google Map was done with the price of the airline tickets of All Nippon Airways (ANA). A significant result was obtained statistically by α = 15562.75 and β = 21.21 with correlation coefficient 0.89 and a coefficient of determination 0.80.

Next, passenger q_{ij} travelling between i and j is considered. Data existing for passengers between existing routes can be determined using data of the Ministry of Land, Infrastructure, Transport, and Tourism. Passenger q_{ij} of the non-existent route is estimated using the gravity model used well in aeronautic demand forecasting. A passenger can be estimated using the following formula and turns into the correlation coefficient 0.806 by a= 0 and b = 2.631×10^{-8}. However, only routes of not less than 300 km of distance can be estimated, and passenger q_{ij} for routes less than 300 km are set to zero.

$$q_{ij} = b\frac{pop_i \cdot pop_j}{d_{ij}{}^{a}} \tag{16}$$

pop : quota population of each airport

Data used as explanatory variables are the quota population of each arrival-and-departure airport and the distance in a straight line d_{ij} between airports. Because the quota population of each airport is needed here, we presume that the following methods are used. In this paper, a Voronoi diagram is drawn by setting each airport to the generatrix (node): the population of each area residing with a nearby airport is used most. The quota population (Table 1) of each airport was calculated by the figure of the area divisions and population data are given by Asahi Shimbun Publications. Data of the existing route are obtained using data provided by the Ministry of Land, Infrastructure, Transport and Tourism. Data estimated using the gravity model are used only for the non-existent route.

Each datum used for a cost function is set to a jet fuel value rank FP = 50 yen/liter, an aviation fuel tax TAX= 26 yen/liter, the landing fee B = 400,000 yen, and weight per passenger PW = 100 kg.

4.2 Optimization simulation

we simulate the case of one kind of aircraft. Data of the aircraft are computed from the average value of the main aircraft (Table 2). The determined optimal network by the simulation is presented in Fig. 3 – Fig. 6.

When the fare discount rate (1 - δ_1) and the passenger decrease rate (1 - δ_2) become small, the optimum network is clarified as centralized on Tokyo International Airport (Haneda). The fare discount rate (1 - δ_1) and the traveler decrease rate (1 - δ_2) become small as $\delta = \delta_1\delta_2$ becomes large. That relation corresponds to the previous work clearly to make the network excessively concentrated by growing of δ. The change to Fig. 4 from Fig. 3 occurs when the passenger decrease rate (1 - δ_2) becomes small. This change shows that the direct flight from the main island of Japan to Naha Airport is decreasing by two routes. The change to Fig. 5 from Fig. 3 changes when the passenger decrease rate (1 – δ_1) becomes small. This change shows that the direct flights from the main island of Japan to Naha Airport are decreasing by four routes, and that numerous direct flights between the airports in the main island of Japan are also decreasing. Furthermore, the change to Fig. 6 from Fig. 3 occurs when both the fare discount rate (1 - δ_1) and the passenger decrease rate (1 - δ_2) become small. The change to Fig. 5 from Fig. 3 and a difference are not apparent. Consequently, among the fare discount rate (1 - δ_1) and the passenger decrease rate (1 - δ_2), the fare discount rate (1 - δ_1) has a stronger influence on the optimal network. If the fare discount rate (1 - δ_1) is small, then because the airline can gain greater profits also from a transit flight (indirect link), it will be expected that the number of a direct flights will decrease. However, if it is expected that there is negative correlation in the fare discount rate and the passenger decrease rate, then the passenger decrease rate will become large when the fare discount rate becomes small.

Moreover, graph values of all the optimal network of Fig. 3 – Fig. 6 have exceeded graphed values of the existing network. Compared with the existing network, the optimal network by a simulation has a tendency with few routes in the graph of the optimal network. The tendency is notably apparent for the route involving Ishigaki Airport. Although a direct flight exists between each airport and Ishigaki Airport in the main island of Japan in the existing network, only the transit flight through Naha Airport exists in the optimal network graph. Therefore, cost cutting is possible by losing a direct flight with a great distance between Ishigaki Airport and the airport in the main

Fig. 3. Optimal network ($\delta_1 = 0.50$, $\delta_2 = 0.50$).

Fig. 5. Optimal network ($\delta_1 = 0.90$, $\delta_2 = 0.50$).

Fig. 4. Optimal network ($\delta_1 = 0.50$, $\delta_2 = 0.90$).

Fig. 6. Optimal network ($\delta_1 = 0.90$, $\delta_2 = 0.90$).

island of Japan. However, in this experiment, the passengers of a non-existent route are estimated using the population around an airport, and it cannot be considered that Ishigaki Island is a tourist resort. The route between Haneda Airport and Ishigaki Airport is known for a seat-occupancy rate being high, and probably, it is not realistic to abolish the route. Regarding this point, the seat-occupancy rate is included in a cost function, and improvement with a different seat-occupancy rate for every route can be considered.

5 Conclusion and Future work

As described in this paper, an airline network is modeled based on Jackson and Wolinsky's (1996) connections model. Moreover, the optimal network in an existing airline network is found using the airline network model. Results of simulations show that because a realistic network identified, the possibility exists that a connections model can be used as a framework for airline network analysis. Although Jackson reported that the connections model can be applied as a framework of various network analyses, few examples have been presented in which a connections model is actually

TABLE II
Aircraft data

	Boeing 747-400D	Boeing 777-300	Boeing 777-200	Average	Used data
Number of seats 2 class	524	451	400	458.33	450
Maximum fuel capacity (Unit: liter)	216840	171160	117335	168445.00	150000
Longest cruising range (Unit: km)	13450	11135	9696	11427.00	10000
Operating empty weight (Unit: kg)	181000	160000	138000	159666.67	150000

(Source: Japan Aircraft Development Corp.)

applied to actual network analysis. This paper has contributed a research example using a connections model. The G-PBIL algorithm that extended PBIL was used for optimization of an airline network. In the simulation, a network that is more efficient than the existing network can be found. When the fare discount rate and the traveler decrease rate were small, the optimum network was shown to centralize to Tokyo International Airport (Haneda).

Future works will expand the simulated range. With the G-PBIL algorithm, although improvement in search performance was sought using a local search together to PBIL, the time which search takes has also increased. The search time by local search increases as the network scale becomes large, and it becomes difficult to perform a simulation. Therefore, it is necessary to consider the proper balance of the evolutionary computation and the local search in G-PBIL. Moreover, although this simulation of only a domestic flight was performed, the international role of Haneda Airport can be clarified by adding main airports of the world. As a subject for other future work, analysis of the optimal airline network when an airport is closed or built is also possible by application of estimation of passengers using a Voronoi diagram and gravity model.

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