Chaotic Subsystem Come From Glider E^3 of CA Rule 110

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Abstract— *The existence of glider in the evolution space of the one-dimensional cellular automaton rule 110, has important lines of investigation in cellular automata theory such as complex dynamical behavior, self-reproduction, universal computation and so on. This work reveals a subsystem based on the existing glider E^3 under the framework of the symbolic dynamics, and proves that the global map of the rule is chaotic in the sense of both Li-Yorke and Devaney on the subsystem.*

Keywords: cellular automata (CA); chaos; de Bruijn diagram; glider; topologically transitive.

1. Introduction

Cellular automata (CA), introduced by von Neumann in the late 1940s and early 1950s, are a class of spatially and temporally discrete mathematical systems characterized by local interactions and synchronous dynamical evolution [1]. Among the 88 possible unique elementary cellular automata (ECA), rule 110 in Stephen Wolfram's system of identification [2] has been an object of special attention due to the structures or gliders which have been observed in evolution space from random initial conditions.

One of the first investigations about rule 110 was described by Wolfram [3], discovering that rule 110 displays complex behaviors by means of the existence of gliders - a glider is a periodic structure moving into the evolution space - from random initial conditions. Thus Wolfram establishes the conjecture that this rule could perform universal computation.

Lind presented the first classification of gliders in rule 110 in [3] with 13 gliders. Next the first paper dedicated to the analysis of rule 110 is made by Lindgren and Nordahl in [4], where a statistical study and some of the most common behaviors of rule 110 are considered. Cook presented his proof of the universality of rule 110 in a conference which taken place at the Santa Fe Institute in 1998 [5,7,8]. On the other hand, another perspective is reported by McIntosh in [6], analyzing rule 110 as a problem of tiles and applying de Bruijn diagrams for characterizing every glider. In this way, Wolfram presents his book *A New Kind of Science* [2] in 2002. The book explains the features of the gliders and the functionality of a cyclic tag system (CTS) to demonstrate that rule 110 is an elemental universal CA [8]. Since 2004, Martinez and his partners further investigated the types of gliders, their properties and collisions and their representation by tiles [9-11].

With this background, the aim of this paper is to reveal a little complex nature contained in rule 110 under the framework of the symbolic dynamical systems. That is, based on the existing glider E^3, this paper find a subsystem on which the global map of rule 110 is chaotic in the sense of both Li-Yorke and Devaney.

2. Symbolic Dynamics and de Bruijn Diagram

2.1 Symbolic sequence space

Let a be a finite or infinite sequence over $S = \{0, 1\}$ and $I = [i, j]$ be an interval of integers on which a is defined, then denote $a_{[i,j]} = (a_i, \cdots, a_j)$ and $a_{[i,j)} = (a_i, \cdots, a_{j-1})$. A sequence b is said to appear in a, denoted by $b \prec a$, if $b = a_I$ for some interval $I \subseteq \mathbb{Z}$.

A bi-infinite sequence over S is called a configuration, the collection of all configurations is $\Sigma_2 = S^{\mathbb{Z}} = \{(\cdots, x_{-1}, \overset{*}{x}_0, x_1, \cdots) |\ x_i \in S, i \in \mathbb{Z}\}$, and the distance "$d$" is defined by

$$d(x, y) = \sum_{i=-\infty}^{\infty} \frac{|x_i - y_i|}{2^{|i|}}$$

for $x, y \in \Sigma_2$. It is well known that Σ_2 is a Cantor complete metric space. The left-shift map σ_L and right-shift map σ_R are defined by $[\sigma_L(x)]_i = x_{i+1}$ and $[\sigma_R(x)]_i = x_{i-1}$ for any $x \in \Sigma_2, i \in \mathbb{Z}$, respectively, where $[\sigma(x)]_i$ stands for the i-th symbol of $\sigma(x)$, and σ is left-shift or right-shift.

2.2 Truth table of rule 110

By a theorem of Hedlund [12], a map $f : \Sigma_2 \to \Sigma_2$ is a cellular automaton iff it is continuous and commutes with σ. Moreover, for any CA f, (Σ_2, f) defines a dynamical system (Σ_2, f). A subset $X \subseteq \Sigma_2$ is f-invariant if $f(X) \subseteq X$, and strongly f-invariant if $f(X) = X$. If X is a closed and f-invariant, then (X, f) or simply X is called a subsystem of (Σ_2, f) [13].

Each ECA rule can be expressed by a local function [18-22], the logical truth table of rule 110's local function $\hat{f}_{110}$ is shown in Table 1.

Table 1: Logical truth table of rule 110's local function

(x_{i-1}, x_i, x_{i+1})	$\hat{f}_{110}(x_{i-1}, x_i, x_{i+1})$
(0, 0, 0)	0
(0, 0, 1)	1
(0, 1, 0)	1
(0, 1, 1)	1
(1, 0, 0)	0
(1, 0, 1)	1
(1, 1, 0)	1
(1, 1, 1)	0

2.3 SFT and de Bruijn diagram

Let $\mathcal{A}$ be n-sequence set over $S = \{0, 1\}$, and $\Lambda_{\mathcal{A}}$ be the set of the configurations whose any n-subsequence is in $\mathcal{A}$. Then, $(\Lambda_{\mathcal{A}}, \sigma)$ is a subsystem of (Σ_2, σ), where $\Lambda_{\mathcal{A}} = \{x \in \Sigma_2 | \; x_{[i,i+n-1]} \in \mathcal{A}, \forall i \in Z\}$, and $\mathcal{A}$ is said to be the determinative block system of $\Lambda_{\mathcal{A}}$. $(\Lambda_{\mathcal{A}}, \sigma)$ (or simply $\Lambda_{\mathcal{A}}$) is also called the subshift of finite type (SFT) of (Σ_2, σ). Furthermore, $\Lambda_{\mathcal{A}}$ can be described by a finite directed graph, $G_{\mathcal{A}} = \{\mathcal{A}, E\}$, where each vertex is labeled by a sequence in $\mathcal{A}$, and E is the set of edges connecting the vertices in $\mathcal{A}$. The finite directed graph $G_{\mathcal{A}}$ is called the de Bruijn diagram of $\mathcal{A}$ (or $\Lambda_{\mathcal{A}}$). Two vertices $a = (a_0, \cdots, a_{n-1})$ and $b = (b_0, \cdots, b_{n-1})$ are connected by an edge $(a_0, \cdots, a_{n-1}) \to (b_0, \cdots, b_{n-1})$ if and only if $(a_1, \cdots, a_{n-1}) = (b_0, \cdots, b_{n-2})$. One can think of each element of $\Lambda_{\mathcal{A}}$ as a bi-infinite path on the diagram $G_{\mathcal{A}}$. Whereas a de Bruijn Diagram corresponds to a square transition matrix $A = (A_{ij})_{m \times m}$ with $A_{ij} = 1$ if and only if there is an edge from vertex $b^{(i)}$ to vertex $b^{(j)}$, where $m = |\mathcal{A}|$ is the number of elements in $\mathcal{A}$, and i (or j) is the code of the corresponding vertex in $\mathcal{A}$, $i, j = 0, 1, \cdots, m-1$. Thus, $\Lambda_{\mathcal{A}}$ is precisely defined by the transition matrix A.

Remarkably, a $\{0, 1\}$ square matrix A is irreducible if, for any i, j, there exists an n such that $A^n_{ij} > 0$; aperiodic if there exists an n such that $A^n_{ij} > 0$ for all i, j, where A^n_{ij} is the (i, j) entry of the power matrix A^n. If $\Lambda_{\mathcal{A}}$ is a SFT of (Σ_2, σ), then σ is transitive on $\Lambda_{\mathcal{A}}$ if and only if A is irreducible; σ is mixing if and only if A is aperiodic. Equivalently, A is irreducible if and only if for every ordered pair of vertices $b^{(i)}$ and $b^{(j)}$ there is a path in $G_{\mathcal{A}}$ starting at $b^{(i)}$ and ending at $b^{(j)}$ [13-15].

3. Glider E^3 and The Corresponding Subsystem

3.1 Glider E^3 of rule 110

A glider is a compact group of non-quiescent states traveling along CA lattice, and is a periodic structure moving in time [9, 10]. From the viewpoint of symbolic dynamics, a glider can be defined as the evolutionary orbit starting from a special initial configuration in the bi-infinite sequence space Σ_2 [16].

It is known that the speed of glider E^3 of rule 110 is $-4/15$, the lineal volume is 19, and the even and odd number of periodic margin on the left (or right) border in the ether pattern are 3 and 1 respectively [10]. From the viewpoint of symbolic dynamics, the ether pattern and glider can be defined as the evolutionary orbit starting from a special initial configuration [16]. The ether factor of the ether patterns e_r (or e_l) is $a = (1, 1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 1, 0)$, and one of glider factors of E^3 is $b = (1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0)$, i.e., an ether pattern of rule 110 is the evolutionary orbit $Orb_{f_{110}}(a^*) = \{a^*, f_{110}(a^*), f^2_{110}(a^*), \cdots\}$ and glider E^3 is the evolutionary orbit $Orb_{f_{110}}(\bar{x}) = \{\bar{x}, f_{110}(\bar{x}), f^2_{110}(\bar{x}), \cdots\}$ in the CA lattice space, where $a^* = (\cdots, a, a, a, \cdots)$ is a cyclic configuration and $\bar{x} = (\cdots, a, a, a, b, a, a, a, \cdots)$. That the speed of E^3 is $-4/15$ implies this glider shifts to left by 4 bits in every 15 iterations under rule 110, i.e., $f^{15}_{110}(\bar{x}) = \sigma^4_L(\bar{x})$. The ether pattern and glider E^3 are shown in Fig. 1.

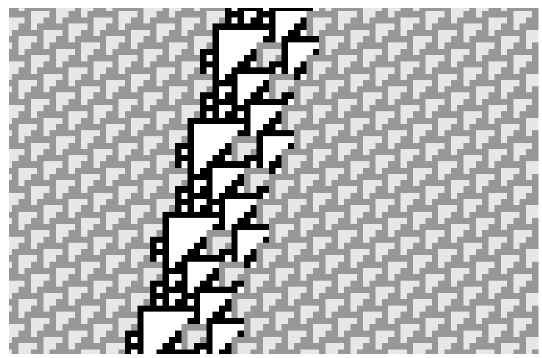

Fig. 1: Ether pattern and glider E^3 of rule 110.

3.2 Subsystem of rule 110

First, one can obtain a 31-sequence set $\mathcal{B} = \{q | q = \bar{x}_{[i,i+30]}, \forall i \in Z\}$ from $\bar{x} = (\cdots, a, a, a, b, a, a, a, \cdots)$, where $a = (1, 1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 1, 0)$ is the ether factor and $b = (1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0)$ is a glider factor of E^3, as

$\mathcal{B} =$

{11111000100110111111000100110111, 1111000100110111111000100110111 1,
1110001001101111110001001101111 1, 1100010011011111100010011011111 0,
1000100110111111000100110111111 00, 0001001101111110001001101111110 00,
0010011011111100010011011111100 01, 0100110111111000100110111111000 10,
1001101111110001001101111110001 00, 0011011111100010011011111100010 01,
0110111111000100110111111000100 11, 1101111110001001101111110001001 10,
1011111100010011011111100010011 01, 0111111000100110111111000100110 10,
0111110001001101111110001001101 0, 1111100010011011111100010011010 0,
1111000100110111111000100110100 1, 1110001001101111110001001101001 1,
1100010011011111100010011010011 1, 1000100110111111000100110100111 1,

0001001101111100010011010011111,
0100110111110001001101001111111,
0011011111000100110100111111101,
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1110011011111000100110111110001,
0010011011111000100110100111111,
1001101111100010011010011111110,
0110111110001001101001111111010,
1011111000100110100111111101011,
1111100010011010011111110101110,
1110001001101001111111010111001,
1000100110100111111101011100110,
0010011010011111110101110011011,
1001101001111111010111001101111,
0110100111111101011100110111110,
1010011111110101110011011111000,
1001111111010111001101111100010,
0111111101011100110111110001001,
1111110101110011011111000100110,
1111010111001101111100010011011,
1101011100110111110001001101111,
0101110011011111000100110111110,
0111001101111100010011011111000,
1100110111110001001101111100010}.

Let $\Lambda_0 = \Lambda_{\mathcal{B}} = \{x \in \Sigma_2 |\ x_{[i,i+30]} \in \mathcal{B}, i \in Z\}$. Since the 15 times iteration of the local function $\hat{f}_{110}$ is a map $\hat{f}_{110}^{15} : S^{31} \to S$, and obviously, $\hat{f}_{110}^{15}(q) = q_{i+4}$ for any $q = (q_{i-15}, \cdots, q_i, \cdots, q_{i+15}) \in \mathcal{B}$. Thus, it follows that $f_{110}^{15}(x) = \sigma_L^4(x)$ for $x \in \Lambda_0$. Furthermore, let

$$\Lambda = \bigcup_{i=0}^{14} f_{110}^i(\Lambda_0).$$

The following propositions can be easily verified.

Proposition 1: Λ is closed f_{110}-invariant set, and $f_{110}^{15}(x) = \sigma_L^4(x)$ for $x \in \Lambda$.

Proposition 2: Λ is a subshift of finite type (SFT) of σ_L.

Let $\mathcal{A}$ be a determinative block system of Λ, then, $\Lambda = \Lambda_{\mathcal{A}}$, where $\mathcal{A}$ is a 31-sequence set consisting of 595 elements. Due to space limitation and simplicity, the decimal code set $D(\mathcal{A})$ of $\mathcal{A}$ is placed in Appendix.

3.3 Chaoticity of rule 110

In the subsection, the chaoticity of rule 110 on $\Lambda_{\mathcal{A}}$ will be revealed.

Proposition 3:
(1) σ_L is topologically transitive on Λ;
(2) f_{110} is topologically transitive on Λ.

Proof: (1) In fact, it can be verified that for every ordered pair of vertices $b^{(i)}$ and $b^{(j)}$ in $\mathcal{A}$ there is a path in the de Bruijn Diagram $G_{\mathcal{A}} = \{\mathcal{A}, E\}$ starting at $b^{(i)}$ and ending at $b^{(j)}$, thus, the transition matrix $A = (A_{ij})_{595\times 595}$ corresponding to $G_{\mathcal{A}}$ is irreducible, so σ_L is topologically transitive on Λ [13, 14].
(2) Similar to [17], the topological transitivity of f_{110} on Λ can be proved. ∎

Proposition 4: The set of periodic points of f_{110}, $P(f) = \{y \in \Lambda |\ \exists\ n > 0, f^n(y) = y\}$, is dense in Λ.

Proof: For any $x \in \Lambda$ and $\epsilon > 0$, there exists a positive integer M (> 15) such that $\sum_{i=M+1}^{\infty} \frac{1}{2^i} < \epsilon/2$, and for $(a_0, \cdots, a_{2M}) = x_{[-M,M]} \prec x \in \Lambda$, it follows that $(a_{2M-30}, \cdots, a_{2M})$, $(a_0, \cdots, a_{30}) \in \mathcal{A}$. Since σ is transitive on Λ, there exists a path from $(a_{2M-30}, \cdots, a_{2M})$ to $(a_0, \cdots, a_{30})$ in $G_{\mathcal{A}} = \{\mathcal{A}, E\}$. Let $\tilde{b} = (a_{2M-30}, \cdots, a_{2M}, b_0, \cdots, b_{k_0}, a_0, \cdots, a_{30})$ be the sequence corresponding to this path. Then, its any 31-subsequences belong to $\mathcal{A}$.

Now, construct a cyclic configuration $y = c^* = (\cdots, c, c, c, \cdots)$, where $c = (a_0, \cdots, a_{2M}, b_0, \cdots, b_{k_0})$. Obviously, $y \in \Lambda$ and $\sigma^m(y) = y$, where $m = |c|$ is the length of c. Thus, $f^{15m}(y) = \sigma^{4m}(y) = y$ and $y_{[-M,M]} = x_{[-M,M]}$, i.e., y is a periodic point of f_{110} and $d(x, y) < \epsilon$. Therefore, the set of periodic points $P(f)$ is dense in Λ. ∎

Proposition 5: The topological entropy of f_{110} is positive.

Proof: The topological entropy of $f_{110}|_\Lambda$ satisfies $ent(f_{110}) \geq ent(f_{110}|_\Lambda) = \frac{4}{15} ent(\sigma_L|_\Lambda) = \frac{4}{15} log(\rho(A)) \approx 0.0179 > 0$, where $\rho(A)$ is the spectral radius of the transition matrix A corresponding to $\mathcal{A}$. ∎

It is well known that positive topological entropy implies chaos in the sense of Li-Yorke [14, 15], and topological transitivity and density of periodic points imply chaos in the sense of Devaney [23, 24]. Thus, one has the interesting result.

Theorem 1: f_{110} is chaotic in the sense of both Li-Yorke and Devaney on Λ.

4. Conclusion

In this paper, it is shown that rule 110 defines a chaotic subsystem on which its global map is topologically transitive, dense periodic points and has positive topological entropy. Thus, it is chaotic in the sense of both Li-Yorke and Devaney. Nevertheless, the complete symbolic dynamical properties of rule 110 are still an open problem. We need to find new ways to uncover its rich and complex dynamics. Conclusively, the result obtained in this paper provides intriguing and valuable clues for researching CA as rule 110.

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Appendix

The determinative block system $\mathcal{A}$ of Λ is a 31-sequence set consisting of 595 elements. For a sequence $a = (a_0, a_1, \cdots, a_{30}) \in \mathcal{A}$, its decimal code is defined as

$$D(a) = \sum_{i=0}^{30} a_i \cdot 2^{30-i}$$

For simplicity, $\mathcal{A}$ is replaced by its decimal code set $D(\mathcal{A})$. $D(\mathcal{A}) =$ {5078093, 10156187, 14664927, 20312375, 29329854, 32525809, 40624751, 40864190, 51437331, 58659708, 65051618, 65171337, 65790575, 81249503, 81728380, 102874662, 103217307, 104736504, 116956130, 117319416, 120713527, 130103236, 130342675, 131581151, 135134285, 141527486, 159510605, 160549965, 162002381, 162335821, 162499006, 163066125, 163263949, 163447093, 163455031, 163456519, 163456583, 163456671, 163456692, 163456736, 163456748, 163456760, 163456762, 163456763, 163456767, 163461335, 163461569, 163462193, 163481393, 163496437, 163510261, 163574221, 205749325, 206434615, 209473009, 225645436, 233912260, 234638833, 238406733, 241332105, 241427055, 242190782, 242252877, 252476493, 260206473, 260685350, 263162302, 270268571, 283054972, 298601969, 319021211, 321099931, 324004763, 324671643, 324998012, 326132251, 326527899, 326894187, 326910063, 326913039, 326913166, 326913343, 326913384, 326913473, 326913497, 326913520, 326913521, 326913524, 326913526, 326913527, 326913535, 326922670, 326923139, 326924387, 326962787, 326992875, 327020523, 327148443, 404686047, 411498651, 412869231, 418946018, 421315807, 444554463, 449889503, 451290872, 451344539, 452500450, 461574367, 464739551, 467669855, 467796863, 467820668, 467821683, 467823101, 467823427, 467824140, 467824333, 467824512, 467824520, 467824521, 467824526, 467824527, 467824545, 467824560, 467824573, 467824636, 467897715, 467901471, 467911455, 468218655, 468459359, 468680543, 469277666, 469703903, 471798652, 476813467, 482664211, 482854111, 484381564, 484505755, 504952987, 520412947, 521370701, 526324604, 534198212, 540537143, 562675238, 566109944, 567049293, 597203938, 638042423, 642199863, 648009527, 649343287, 649996024, 652264503, 653055799, 653788375, 653820127, 653826079, 653826332, 653826687, 653826768, 653826947, 653826995, 653827040, 653827042, 653827043, 653827048, 653827052, 653827055, 653827071, 653845340, 653846279, 653848775, 653925575, 653985751, 654041047, 654296887, 670420465, 755781105, 778203017, 779061694, 797556262, 809372094, 813530189, 822997303, 825738463, 837892036, 842631614, 861724435, 875640772, 889108926, 899779006, 902581745, 902689079, 905000900, 923148734, 929479102, 935339710, 935593726, 935641336, 935643366, 935646202, 935646854, 935648280, 935648666, 935649024, 935649040, 935649043, 935649052, 935649054, 935649091, 935649120, 935649147, 935649272, 935795430, 935802942, 935822910, 936437310, 936918718, 937361086, 938555332, 939407806, 942738929, 943597304, 953626935, 965328422, 965708222, 968763128, 969011511, 993381873, 1009905975,1040022605, 1040266737, 1040825894, 1042298865, 1042679745, 1042695985, 1042718679, 1042723889, 1042735300, 1042738391, 1042741252, 1042741376, 1042741400, 1042741401, 1042741402, 1042741403, 1042741475, 1042741479, 1042741488, 1042741788, 1042742023, 1042742239, 1042743239, 1043912497, 1043972593, 1044132337, 1049047537, 1052649208, 1052898801, 1056437745, 1065991718, 1068396425, 1068468670, 1072811505, 1076280870, 1081074287, 1090004728, 1094173919, 1099460489, 1106327492, 1106637111, 1125350477, 1126110076, 1132219889, 1134098587, 1141308966, 1144505567, 1153497126, 1154016806, 1154743014, 1154909734, 1155274886, 1155373798, 1155465370, 1155469339, 1155470083, 1155470115, 1155470159, 1155470170, 1155470192, 1155470198, 1155470204, 1155470205, 1155470207, 1155472491, 1155472608, 1155472920, 1155482520, 1155490042, 1155496954, 1155528934, 1186564542, 1192945190, 1194407876,

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