Angular power spectrum of scattered radiation in ionospheric plasma with both electron density and magnetic field fluctuations

G. Jandieri[1], N. Zhukova[2], Zh. Diasamidze[1], and M. Diasamidze[1]
[1]Department of Physics, Georgian Technical University, Tbilisi, Georgia
[2]Dynamics of Geophysical fields and Computing Geophysics, TSU M. Nodia Institute of Geophysics, Tbilisi, Georgia

Abstract - *The influence of anisotropy of both electron density and external magnetic field fluctuations on the spatial power spectrum (SPS) of scattered electromagnetic waves is considered in this paper. Stochastic differential equation is obtained for the phase fluctuations using smooth perturbation method taking into account diffraction effects. Second order statistical moments are calculated for arbitrary correlation functions of electron density and external magnetic field fluctuations. Numerical calculations were carried out for anisotropic Gaussian correlation function containing nondimensional anisotropic parameter and the angle of inclination of prolate irregularities with respect to the external magnetic field. SPS of scattered radiation has a pronounced gap caused by electron density fluctuations. The influence of an external magnetic field on a double-peaked shape has been analytically and numerically.*

Keywords: Ionospheric plasma, Anisotropy, Phase fluctuations, Angular power spectrum

1 Introduction

Peculiarities of the electromagnetic waves propagation in randomly inhomogeneous media have been intensively studied [1,2]. However, the large-scale irregularities were considered to be statistically isotropic. In many cases irregularities are anisotropic. Particularly, they are observed in lyotropic crystals with a hexagonal structure [3], in the Earth's ionosphere random plasma inhomogeneities are aligned with the geomagnetic fields [4]. The evolution of the angular distribution of the intensity at light propagation in a randomly unhomogeneous medium with strongly prolated anisotropic irregularities of dielectric permittivity has been investigated in [5,6]. Using the smooth perturbation method it has been shown that the spatial power spectrum (SPS) of multiply scattered waves at oblique illumination of a boundary of a randomly inhomogeneous medium with prolate irregularities by mono-directed incident radiation has a double-peaked shape. Numerical simulation has been carried out by Monte-Carlo method. Second order statistical moments of the SPS in magnetized anisotropic plasma have been investigated in the complex geometrical optics approximation and perturbation method [7-10].

The features of the SPS of multiply scattered radiation in a randomly inhomogeneous anisotropic ionospheric plasma are investigated analytically and numerically taking into account diffraction effects caused by both electron density and external magnetic field fluctuations. The expressions for phase fluctuations of scattered electromagnetic waves in the principle (wave vector of mono-directed incident radiation and external magnetic field are located in this plane) and perpendicular planes are derived using the smooth perturbation method. Correlation functions of the phase fluctuations are calculated for arbitrary correlation functions of fluctuating magnetized plasma parameters. The influence of an external magnetic field on a gap caused due to electron density fluctuations in the ionospheric plasma is considered for the first time in this paper. Numerical calculations are carried out using satellite and remote sensing data.

2 Formulation of the problem

Let us consider the features of the SPS of scattered electromagnetic waves in the anisotropic ionospheric magnetized plasma with both electron density and external magnetic field fluctuations. Initial is the following wave equation:

$$\left(\frac{\partial^2}{\partial x_i \, \partial x_j} - \Delta \delta_{ij} - k_0^2 \, \varepsilon_{ij}(\mathbf{r}) \right) \mathbf{E_j}(\mathbf{r}) = 0 \,. \qquad (1)$$

Wave field we introduce as $E_j(\mathbf{r}) = E_{0j} \exp(\varphi_1 + \varphi_2 + . + i k_\perp y + i k_0 z) \, (k_\perp << k_0)$. If electromagnetic wave propagates along z axis and the vector of an external magnetic field lies in the coordinate plane ($\mathbf{k} \parallel z$, $< \mathbf{H_0} > \in yz$), components of the second-rank tensor of collisionless magnetized plasma have the following form [11]:

$$\varepsilon_{xx} = 1 - \frac{\mathrm{v}}{1-u}\ , \quad \varepsilon_{yy} = 1 - \frac{\mathrm{v}(1-u\sin^2\alpha)}{1-u}\ ,$$

$$\varepsilon_{zz} = 1 - \frac{\mathrm{v}(1-u\cos^2\alpha)}{1-u}, \quad \varepsilon_{xy} = -\varepsilon_{yx} = i\,\frac{\mathrm{v}\sqrt{u}\cos\alpha}{1-u}\ ,$$

$$\varepsilon_{yz} = \varepsilon_{zy} = \frac{u\,\mathrm{v}\sin\alpha\cos\alpha}{1-u}, \quad \varepsilon_{xz} = -\varepsilon_{zx} = -i\,\frac{\mathrm{v}\sqrt{u}\sin\alpha}{1-u} \tag{2}$$

where α is the angle between the vectors $\mathbf{k}$ and $\mathbf{H_0}$; $\varepsilon_{xy} = i\,\tilde{\varepsilon}_{xy}$, $\varepsilon_{xz} = -i\,\tilde{\varepsilon}_{xz}$, $u = (eH_0/mc\omega)^2$, $\mathrm{v} = \omega_p^2/\omega^2$ are the magneto-ionic parameters, $\omega_p = (4\pi N e^2/m)^{1/2}$ is the plasma frequency, $\Omega_H = eH_0/mc$ is the electron gyrofrequency. Dielectric permittivity of a turbulent magnetized plasma is a second rank tensor, which is random function of a spatial coordinates $\varepsilon_{ij}(\mathbf{r}) = \varepsilon_{ij}^{(0)} + \varepsilon_{ij}^{(1)}(\mathbf{r})$, $|\varepsilon_{ij}^{(1)}(\mathbf{r})| << 1$. First component represents zero-order approximation, second one containes fluctuations of both electron density and external magnetic field fluctuations of the ionospheric plasma which are random functions of the spatial coordinates: $\mathrm{v}(\mathbf{r}) = \mathrm{v}_0[1+n_1(\mathbf{r})]$, $u(\mathbf{r}) = u_0\,[1+ +2h_1(\mathbf{r})]$.

In a zero-order approximation we have the following wave equation

$$\left[\frac{\partial^2\varphi_0}{\partial x_i\,\partial x_j} + \frac{\partial\varphi_0}{\partial x_i}\frac{\partial\varphi_0}{\partial x_j} + (k_\perp^2 + k_0^2)\,\delta_{ij} - k_0^2\,\varepsilon_{ij}^{(0)}\right]E_{0j} = 0\ , \tag{3}$$

containing the set of three algebraic equations for the E_{0j} regular field components:

$$\left(\varepsilon_{xx}^{(0)} - 1 - \frac{k_\perp^2}{k_0^2}\right)E_{0x} + i\,\tilde{\varepsilon}_{xy}^{(0)}E_{0y} - i\,\tilde{\varepsilon}_{xz}^{(0)}E_{0z} = 0\ ,$$

$$i\,\tilde{\varepsilon}_{xy}^{(0)}E_{0x} + (1-\varepsilon_{yy}^{(0)})E_{0y} - \left(\frac{k_\perp}{k_0} + \varepsilon_{yz}^{(0)}\right)E_{0z} = 0\ ,$$

$$i\,\tilde{\varepsilon}_{xz}^{(0)}E_{0x} + \left(\varepsilon_{yz}^{(0)} + \frac{k_\perp}{k_0}\right)E_{0y} - \left(\frac{k_\perp^2}{k_0^2} - \varepsilon_{zz}^{(0)}\right)E_{0z} = 0\ . \tag{4}$$

Solution of determinant imposes the restriction on the parameter $\mu = k_\perp/k_0$:

$$(2-\varepsilon_{yy})\,\mu^4 + 2\varepsilon_{yz}\,\mu^3 + (2 - 2\varepsilon_{xx} - \varepsilon_{yy} - \varepsilon_{zz} + \varepsilon_{xx}\,\varepsilon_{yy} +$$

$$+\varepsilon_{yy}\,\varepsilon_{zz} - \varepsilon_{yz}^2 - \tilde{\varepsilon}_{xy}^2)\,\mu^2 + 2\left[\tilde{\varepsilon}_{xy}\,\tilde{\varepsilon}_{xz} + \varepsilon_{yz}(1-\varepsilon_{xx})\right]\mu\ +$$

$$+\Big[\varepsilon_{zz}(\varepsilon_{xx} - \varepsilon_{xx}\varepsilon_{yy} - 1 + \varepsilon_{yy}) + 2\tilde{\varepsilon}_{xy}\,\tilde{\varepsilon}_{xz}\,\varepsilon_{yz} + \tilde{\varepsilon}_{xz}^2(\varepsilon_{yy}-1) +$$

$$+\varepsilon_{yz}^2(\varepsilon_{xx}-1) + \tilde{\varepsilon}_{xy}^2\,\varepsilon_{zz}\Big] = 0\ . \tag{5}$$

Taking into account that fluctuations of the complex phase are of the order $\varphi_1 \sim \varepsilon_{ij}^{(1)}$, $\varphi_2 \sim \varepsilon_{ij}^{(1)\,2}$ and the well known conditions characterizing the smooth perturbation method:

$$\left|\frac{\partial\varphi_1}{\partial z}\right| << k_0\,|\varphi_1|\ , \quad \left|\frac{\partial^2\varphi_1}{\partial z^2}\right| << k_0\left|\frac{\partial\varphi_1}{\partial z}\right|\ ,$$

$$\left|\frac{\partial\varphi_2}{\partial z}\right| << k_0\,|\varphi_2|\ , \quad \left|\frac{\partial^2\varphi_2}{\partial z^2}\right| << k_0\left|\frac{\partial\varphi_2}{\partial z}\right|,$$

in the first approximation we obtain:

$$\left[\frac{\partial^2\varphi_1}{\partial x_i\,\partial x_j} + \frac{\partial\varphi_0}{\partial x_i}\frac{\partial\varphi_1}{\partial x_j} + \frac{\partial\varphi_1}{\partial x_i}\frac{\partial\varphi_0}{\partial x_j} - \delta_{ij}\left(\Delta_\perp + 2i\,k_\perp\frac{\partial\varphi_1}{\partial y} + 2i\,k_0\frac{\partial\varphi_1}{\partial z}\right) - k_0^2\,\varepsilon_{ij}^{(0)}\right]E_{0j} = 0\ . \tag{6}$$

where $\Delta_\perp = (\partial^2\varphi_1/\partial x^2) + (\partial^2\varphi_1/\partial y^2)$ is the transversal Laplasian.

Two-dimensional Fourier transformation for the phase fluctuations is

$$\varphi_1(x,y,z) = \int_{-\infty}^{\infty} d\,k_x \int_{-\infty}^{\infty} d\,k_y\ \psi(k_x,k_y,z)\exp(i\,k_x x + i\,k_y y)\ ,$$

For $i = x$ component from equation (6) we obtain differential equation for two-dimensioanl spectral component:

$$\frac{\partial\psi}{\partial z} + \frac{i}{k_x\dfrac{E_{0z}}{E_{0x}} - 2k_0}\left[k_x(k_y+k_\perp)\frac{E_{0y}}{E_{0x}} + k_x\,k_0\frac{E_{0z}}{E_{0x}} - k_y(k_y+2k_\perp)\right]\psi = -i\,\frac{k_0^2}{k_x\dfrac{E_{0z}}{E_{0x}} - 2k_0}\ . \tag{7}$$

The relations of the mean electric field components are determined by the well-known formulae [11] $(E_{0y}/E_{0x}) = i\,\mathrm{P}_j$, $(E_{0z}/E_{0x}) = i\,\Gamma_j$, minus sign and index $j=1$ correspond to the extraordinary wave, plus sign and index $j=2$ - to the ordinary wave; the polarization coefficients are [11]:

$$\mathrm{P}_j = \frac{2\sqrt{u}\,(1-\mathrm{v})\cos\alpha}{u\sin^2\alpha \pm \sqrt{u^2\sin^4\alpha + 4u(1-\mathrm{v})^2\cos^2\alpha}}$$

$$\Gamma_j = -\frac{\mathrm{v}\sqrt{u}\sin\alpha + \mathrm{P}_j\,u\,\mathrm{v}\sin\alpha\cos\alpha}{1-u-\mathrm{v}+u\,\mathrm{v}\cos^2\alpha}\ , \tag{8}$$

In general, ordinary and extraordinary waves in collisionless magnetized plasma are elliptically polarized.

Transverse correlation function of a scattered field has the following form [6] $W_{EE^*}(\boldsymbol{\rho}) = < E(\mathbf{r}) E^*(\mathbf{r}+\boldsymbol{\rho}) >$ taking into account that the observation points are spaced apart at a small distance $\boldsymbol{\rho} = \{\rho_x, \rho_y\}$:

$$W_{EE^*}(\boldsymbol{\rho}, k_\perp) = E_0^2 \exp(-i\rho_y k_\perp) \exp\left\{ \mathrm{Re}\left[\frac{1}{2}\left(< \varphi_1^2(\mathbf{r}) > + \right.\right.\right.$$

$$\left.\left.\left. + < \varphi_1^{2*}(\mathbf{r}+\boldsymbol{\rho}) > \right) + < \varphi_1(\mathbf{r})\, \varphi_1^*(\mathbf{r}+\boldsymbol{\rho}) > + 2 < \varphi_2 > \right]\right\}, \quad (9)$$

where E_0^2 is the intensity of an incident radiation.

SPS of a scattered field in case of incident plane wave $W(k', k_\perp)$ is easily calculated by Fourier transform of the transversal correlation function [1,2].

$$W(k', k_\perp) = \int_{-\infty}^{\infty} d\rho_y\, W_{EE^*}(\rho_y, k_\perp) \exp(i k' \rho_y). \quad (10)$$

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2.1 Second order statistical moments of the phase fluctuations

In these notations two-dimensional spectral component of the phase fluctuation of scattered electromagnetic field (7) in the first approximation satisfies the stochastic differential equation:

$$\frac{\partial \psi}{dz} + \frac{i d_1 - d_2}{\Gamma_j k_x + 2 i k_0} \psi(k_x, k_y, z) = -\frac{k_0^2}{\Gamma_j k_x + 2 i k_0}$$

$$\left\{ \varepsilon_{xx}^{(1)}(k_x, k_y, z) - \left[\mathrm{P}_j\, \tilde{\varepsilon}_{xy}^{(1)}(k_x, k_y, z) - \Gamma_j\, \tilde{\varepsilon}_{xz}^{(1)}(k_x, k_y, z) \right] \right\} \quad (11)$$

where: $d_1 = k_x (k_y + k_\perp) \mathrm{P}_j + k_0 k_x \Gamma_j$, $d_2 = k_y (k_y + 2 k_\perp)$. The solution of this equation satisfying the boundary condition has the following form

$$\psi(k_x, k_y, z) = i\, \frac{k_0}{2} \int_{-\infty}^{\infty} dz' \left\{ \varepsilon_{xx}^{(1)}(k_x, k_y, z') - \right.$$

$$\left. - \left[\mathrm{P}_j\, \tilde{\varepsilon}_{xy}^{(1)}(k_x, k_y, z') - \Gamma_j\, \tilde{\varepsilon}_{xz}^{(1)}(k_x, k_y, z') \right] \right\} \cdot$$

$$\cdot \exp\left[-\frac{d_2 - i d_1}{\Gamma_j k_x + 2 i k_0} (L - z') \right]. \quad (12)$$

Taking into account that: $< \mathrm{T}_{\alpha\beta}(\boldsymbol{\kappa}, z')\, \mathrm{T}_{\gamma\delta}(\boldsymbol{\kappa}', z'') > = = W_{\alpha\beta,\gamma\delta}(\boldsymbol{\kappa}, z' - z'')\, \delta(\boldsymbol{\kappa} + \boldsymbol{\kappa}')$ and changing the variables: $z' - z'' = \rho_z$, $z' + z'' = 2\eta$, second order statistical moments of phase fluctuations of scattered electromagnetic waves for arbitrary correlation function of electron density fluctuations are finally expressed as:

$$< \varphi_1^2(\mathbf{r}) > = \frac{\pi k_0^2}{2} \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y\, \frac{G_1 + i G_2}{G_1^2 + G_2^2} \left[V_{xx,xx} + V_{xy,xy} + \right.$$

$$\left. + V_{xz,xz} + 2 (V_{xx,xz} - V_{xx,xy} - V_{xy,xz}) \right] \left\{ 1 - \exp\left[(G_1 - i G_2) L \right] \right\} \quad (13)$$

$$< \varphi_1^{*\,2}(\mathbf{r}+\boldsymbol{\rho}) > = \frac{\pi k_0^2}{2} \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y\, \frac{G_1 - i G_2}{G_1^2 + G_2^2} \left[V'_{xx,xx} + \right.$$

$$\left. + V'_{xy,xy} + V'_{xz,xz} + 2 (V'_{xx,xz} - V'_{xx,xy} - V'_{xy,xz}) \right] \cdot$$

$$\cdot \left\{ 1 - \exp\left[(G_1 + i G_2) L \right] \right\}, \quad (14)$$

$$< \varphi_1(\mathbf{r})\, \varphi_1^*(\mathbf{r}+\boldsymbol{\rho}) > = \frac{\pi k_0^2 L}{2} \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \left[V''_{xx,xx} + V''_{xy,xy} + \right.$$

$$\left. + V''_{xz,xz} + 2 (V''_{xx,xz} - V''_{xx,xy} - V''_{xy,xz}) \right] \exp(-i k_x \rho_x - i k_y \rho_y) \quad (15)$$

where: $V_{\alpha\beta,\gamma\delta} \equiv V_{\alpha\beta,\gamma\delta}(k_x, k_y, i G_3 - G_4)$, $V''_{\alpha\beta,\gamma\delta} \equiv V''_{\alpha\beta,\gamma\delta}\left(k_x, k_y, -(d_1 \Gamma_j k_x + 2 d_2 k_0)/4 k_0^2\right)$, $V'_{\alpha\beta,\gamma\delta} \equiv V'_{\alpha\beta,\gamma\delta}(k_x, k_y, -i G_3 - G_4)$, indices denote the product of fluctuating terms of the second rank tensors;

$$G_1 = \frac{1}{k_0^2} (\Gamma_j k_\perp - P_j k_0) k_x k_y,$$

$$G_2 = \frac{1}{2 k_0^2} \left[\Gamma_j (P_j k_\perp + \Gamma_j k_0) k_x^2 + 2 k_0 k_y^2 \right],$$

$$G_3 = \frac{1}{4 k_0^2} (2 \Gamma_j k_0^2 + 2 P_j k_0 k_\perp - \Gamma_j k_y^2) k_x,$$

$$G_4 = \frac{1}{4 k_0^2} (P_j \Gamma_j k_x^2 + 4 k_0 k_\perp) k_y.$$

For $i = x$ component from equation (1) we obtain stochastic differential equation in the second approximation

$$i P_j \frac{\partial^2 \varphi_2}{\partial x \partial y} + i \Gamma_j \frac{\partial^2 \varphi_2}{\partial x \partial z} - (k_\perp P_j + k_0 \Gamma_j) \frac{\partial \varphi_2}{\partial x} - \frac{\partial^2 \varphi_2}{\partial y^2} -$$

$$-2 i k_\perp \frac{\partial \varphi_2}{\partial y} - 2 i k_0 \frac{\partial \varphi_2}{\partial z} = -\, i P_j \frac{\partial \varphi_1}{\partial x} \frac{\partial \varphi_1}{\partial y} + \left(\frac{\partial \varphi_1}{\partial y} \right)^2, \quad (16)$$

By Fourier transform first term of the right part of equation (16) can be written as:

$$<\frac{\partial\varphi_1}{\partial x}\frac{\partial\varphi_1}{\partial y}> = \frac{\pi k_0^2}{4}\int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \ \frac{k_x k_y}{G_1^2+G_2^2}\Big[V_{xx,xx} +$$

$$+V_{xy,xy}+V_{xz,xz}+2(V_{xx,xz}-V_{xx,xy}-V_{xy,xz})\Big]$$

$$\cdot\left\{ 1-\exp\left[(G_1 - i\,G_2)L \right]\right\}. \qquad (17)$$

Solution of equation (16) is expressed as:

$$\mathrm{Re}<\varphi_2(\mathbf{r})> = \mathrm{Re}\ \frac{\pi k_0}{4}\int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \ \frac{1}{G_1^2+G_2^2}\cdot$$

$$\cdot\left\{ L\,(A_1 + i\,B_1) + \frac{1}{G_1^2+G_2^2}\left[(A_1 G_1 - B_1 G_2) + \right.\right.$$

$$\left.\left. + i\,(B_1 G_1 + A_1 G_2)\right] - \frac{1}{G_1^2+G_2^2}(A_2 + i\,B_2)\exp(G_1 L)\right\}\cdot$$

$$\left[V_{xx,xx}+V_{xy,xy}+V_{xz,xz}+2(V_{xx,xz}-V_{xx,xy}-V_{xy,xz})\right] \quad (18)$$

where: $A_1 = -(P_j\,G_1\,k_x\,k_y + G_2\,k_y^2)$, $A_2 = (A_1G_1 - B_1G_2)\cdot$ $\cos(G_2L) + (B_1G_1 + A_1G_2)\sin(G_2L)$, $B_1 = G_1\,k_y^2 - P_j\,G_2\,k_x\,k_y$, $B_2 = (B_1G_1 + A_1G_2)\cos(G_2L) - (A_1G_1 - B_1G_2)\sin(G_2L)$.

In the absence of an external magnetic field ($H_0 = 0$, $u_0 = 0$), from equation (8) follows: $P_j = \Gamma_j = 0$, $d_1 = 0$, and (12)-(15) coincide with [6].

2.2 Numerical calculations

In analytical and numerical calculations we will use anisotropic Gaussian correlation function of electron density fluctuation [12] for investigation of the influence of electron density and external magnetic field fluctuations on evolution of the SPS

$$W_n(k_x,k_y,k_z) = \sigma_n^2\,\frac{l_\perp^2\,l_\parallel}{8\pi^{3/2}}\ \exp\left(-\frac{k_x^2\,l_\perp^2}{4} - p_1\frac{k_y^2\,\overline{l}^{\,2}}{4} - \right.$$

$$\left. -p_2\frac{k_z^2\,l_\parallel^2}{4} - p_3\,k_y\,k_z\,l_\parallel^2\right). \qquad (19)$$

This function is characterized by anisotropy factor of irregularities $\chi = l_\parallel / l_\perp$ (ratio of longitudinal and transverse linear scales of plasma irregularities with respect to the external magnetic field) and the inclination angle of prolate irregularities with respect to the external magnetic field γ_0.

$p_1 = 1 + (1-\chi^2)^2\sin^2\gamma_0\cos^2\gamma_0/\chi^2$, $p_2 = (\sin^2\gamma_0 +$ $+\chi^2\cos^2\gamma_0)/\chi^2$, $p_3 = (1-\chi^2)\sin\gamma_0\cos\gamma_0/2\chi^2$, $\overline{l} = l_\parallel\left(\sin^2\gamma_0 + \chi^2\cos^2\gamma_0\right)^{-1/2}$. In isotropic case ($\chi = 1$) we have: $p_1 = p_2 = 1$, $p_3 = 0$; at $\gamma_0 = 0^0$: $p_1 = 1/\chi^2$, $p_2 = 1$, $p_3 = 0$.

We investigate the influence of electron density fluctuations on the SPS of scattered radiation in turbulent magnetized plasma ($H_0 \neq 0$). At $T = k_0\,l_\parallel >> 1$ using the saddle point method, we obtain:

$$<\varphi_1^2(\mathbf{r})> + <\varphi_1^{*\,2}(\mathbf{r}+\boldsymbol{\rho})> = -\frac{\sigma_n^2\,\Omega_1}{8\sqrt{2}}\frac{T^2}{\chi^2}$$

$$\int_{-\infty}^{\infty} ds\ \frac{1}{s^2\sqrt{b_1}}\ \sin(s^2 k_0 L)\exp(-T^2 b_2),$$

$$<\varphi_1(\mathbf{r})\ \varphi_1^*(\mathbf{r}+\boldsymbol{\rho})> = \frac{\sigma_n^2\,\Omega_1}{16}\frac{T^2\,k_0\,L}{\chi^2}\int_{-\infty}^{\infty} ds\ \frac{1}{\sqrt{b_6}}$$

$$\exp\left\{-\frac{T^2}{4}\left[p_1\,s^2 + \frac{1}{4}p_2(s^2+2s\mu)^2 + 2p_3(s^3+2s^2\mu)\right]\right.$$

$$\left. - i\eta s\right\},$$

$$\mathrm{Re}<\varphi_2(\mathbf{r})> = -\frac{\sigma_n^2\,\Omega_1}{32}\frac{T^2\,k_0\,L}{\chi^2}\int_{-\infty}^{\infty} ds\ \frac{1}{\sqrt{b_1}}$$

$$\cdot\left[1 - \frac{1}{k_0\,L\,s^2}\sin(k_0\,L\,s^2)\right]\exp(-T^2\,b_2)\ , \qquad (20)$$

where: $b_1 = \dfrac{1}{4\chi^2} - \dfrac{1}{4}p_2(m_3^2 - 2m_4\,\mu s) + p_3\,m_4\,s$, $x = \dfrac{k_x}{k_0}$,

$$\Omega_1 = \frac{\mathrm{v}_0}{(1-u_0)^2}\left[1 + u_0 - 2\sqrt{u_0}(\sin\alpha - \cos\alpha + \sqrt{u_0}\sin\alpha\cos\alpha\right]$$

$s = \dfrac{k_y}{k_0}$, $b_6 = \dfrac{1}{4}\left(\dfrac{1}{\chi^2} + 2\,p_2\,m_5\,m_6 + 4p_3\,m_5\,s\right)$, $\eta = k_0\,\rho_y$,

$B_0 = \sqrt{\pi}\,\sigma_n^2\,\mathrm{v}_0^2\,T\,k_0\,L/4$, $m_3 = \dfrac{1}{4}(2\Gamma_j + 2\mathrm{P}_j\,\mu - \Gamma_j\,s^2)$,

$m_4 = \dfrac{1}{4}\mathrm{P}_j\Gamma_j$, $m_5 = \dfrac{1}{4}\left[(s+\mu)\mathrm{P}_j + \Gamma_j\right]\Gamma_j$,

$m_6 = \dfrac{1}{2}\ (s^2 + 2s\,\mu)$.

Numerical calculations were carried out for 0.1 MHz and 40 MHz at $\alpha = 15^0$. The solution of the dispersion equation (5) yields the roots: $\mu = 0.395$, for 0.1 MHz and $\mu = 0.114$, for 40 MHz.

The curves in Figure 1 illustrate the dependence of normalized correlation function of scattered electromagnetic field versus non-dimensional parameter η for $T = 200$ and are normalized on their maximum value. Second maxima on the solid and dotted lines correspond $\eta = 52$ and $\eta = 16$, respectively. Next maxima at 0.1 MHz appear at $\eta =$ 32, 48, 64 (periodical oscillations). Increasing parameter η normalized correlation functions rapidly attenuates.

Figure 2 presents the dependence of the SPS of scattered field versus nondimensional parameter k. Numerical calculations show that for 0.1 MHz (left figure), at $H_0 = 0$, the gap arises due to electron density fluctuations. First and second maxima correspond $k = 0.355$ and $k = 0.1$, respectively; gap appears at $k = 0.194$. In magnetized turbulent plasma ($H_0 \neq 0$) two pronounces maxima arise at $k = 0.38$ and $k = 0.14$; and two gaps at $k = 0.37$ and $k = 0.42$. For 40 MHz (right figure) at $H_0 = 0$ first maximum arise at $k = 0.11$, and next two maxima at $k = 0.033$ and $k = 0.189$. First two gaps appear at $k = 0.072$ and $k = 0.163$; next two gaps at $k = 0.007$ and $k = 0.228$.

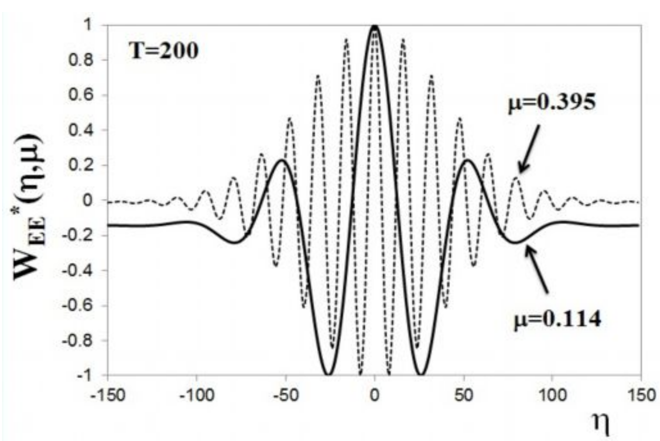

Figure 1. Dependence of normalized correlation function of scattered field $W_{EE^*}(\eta, \mu)$ versus distance between two observation points $\eta = k_0 \rho_y$ at different values of the parameter μ. Dotted line corresponds 0.1 MHz, solid line 40 MHz.

At $H_0 \neq 0$ first and other two maxima arise at $k = 0.06$ and $k = 0.02$, $k = 0.08$, respectively. First gap appears at $k = 0.03$, second one at $k = 0.795$.

It should be emphases that "double-hump" shape of the SPS caused by electron density fluctuations in turbulent plasma without external magnetic field are more pronounced than in magnetized plasma at $H_0 \neq 0$. Numerical analyses show that neglecting diffraction effects, i.e. neglecting the term $k_y^2 / 2k_0^2$ in the arguments of 2D spectrum (13)-(15) or in set of equations (20), "double-humping" effect in the SPS disappears.

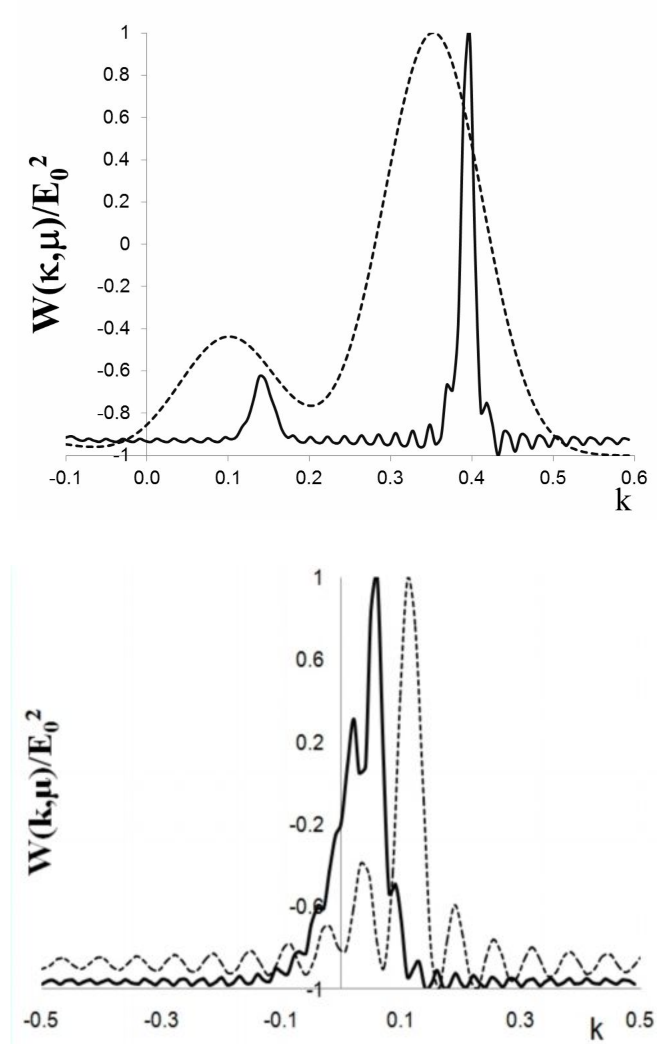

Figure 2. Dependence of SPS (10) versus k. Left figure corresponds to 0.1 MHz at: $T = 2500$, $\mu = 0.395$, $\chi = 130$, $\gamma_0 = 15^0$, $\xi = 1$, $k_0 = 2.8$ km^{-1}, $B_0 = 6$. Right figure corresponds 40 MHz at: $T = 500$, $\mu = 0.114$, $\chi = 150$, $\gamma_0 = 5^0$, $\xi = 1$, $k_0 = 840$ km^{-1}, $B_0 = 4$. Dotted line denotes $H_0 = 0$, solid line $H_0 \neq 0$.

3 Conclusions

Numerical calculations show that for anisotropic Gaussian correlation function second-order statistical moments are nonlinear functions of wave vectors. For electron density fluctuations spatial power spectrum has a pronounced gap along a direction of wave propagation and a double-peaked shape. However, external magnetic field fluctuations can lead to the generation of different nonlinear effects. On the basis of the proposed theory some observable nonlinear effects of the ionospheric plasma can be interpreted and (or) predicted using different correlation functions of fluctuating magnetized plasma parameters taking into account satellite and remote sensing data.

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